1. (a) Write an equation for the perimeter, and then divide it by 2.
   (b) The second equation is "area equals 15".
   (c) Rearrange to $y = \frac{15}{x}$ and substitute for y.
   (d) Solve by factorising. Since the question asks for lengths, give your answers in cm.

2. Multiply both sides by $2x + 1$, then gather the x's together on one side.

3. (a) Use Pythagoras to find each side length.
   (b) Subtract the total area of the three unshaded triangles from the area of the square.

4. (a) Rewrite in simpler algebra, e.g. $b \times c = bc$, then group like terms. Give careful thought to BIDMAS: the order of operations is brackets, indices, division, multiplication, addition then subtraction.
   (b) Expand the brackets, taking care with minus signs. You'll get six terms, of which four will cancel. Take a common factor out of the other two.

5. (a) Complete the square twice, once for the x terms x^2+4x and once for the y terms y^2-6y. Then gather the constant terms.
   (b) Pythagoras tells us that the circle $(x-a)^2 + (y-b)^2 = r^2$ has centre (a, b) and radius r.

6. The squares are not relevant to the logic, and can be ignored. Consider the possibility that only one of x or y is zero.

7. First, find the gradient m with a "rise over run" $\frac{\Delta y}{\Delta x}$ triangle. Then, substitute into the formula $y - y_1 = m(x - x_1)$ and simplify.

8. A and B are independent, so the probability of B doesn't change with information about A. So $\mathbb{P}(B \mid A)$ and $\mathbb{P}(B \mid A')$, i.e. the probability of B conditioned on whether A has happened or not, are both 0.1. Draw a tree diagram conditioned on A: the second set of branches should be identical.
   (a) For the intersection $A \cap B$, there is only one successful branch.
   (b) For the union $A \cup B$, there are three successful branches.

9. Notice that the numerators are negatives of each other, and likewise the denominators. Cancelling the minus signs gives squares, which can then be square rooted.

10. "Non-compound" means the addition of the same amount every month. So, this problem requires no consideration of scale factors, other than the fact that doubling requires an increase of 1 on top of the original quantity 1.

11. The equations are $a - b = 100$, $ab = 5964$. Using substitution, these generate a quadratic equation.

12. (a) Often, giving an "explanation" in mathematics boils down to pointing out an established fact or theorem. Here, that fact concerns tangent and radius.
    (b) In the right-angled triangle formed by the light at sunset, label the distance the light travels through the atmosphere as d. The other two sides are 6370 and 6470. Use Pythagoras to find d, and show that it is a little over 1100.

13. List them alphabetically, starting with AABB and ending up with BBAA.

14. (a) The factor theorem states that $x = a$ is a root of a polynomial $f(x)$ if and only if (iff) $(x - a)$ is a factor. So, substitute $x = 4$ to form an equation in k.
    (b) Substitute k, and factorise the quadratic, noting that $(x - 4)$ should be a factor.

15. You don't need to solve simultaneously. Consider the gradients of the lines.

16. A fraction is zero only if its numerator is zero:
$$\frac{f(x)}{g(x)} = 0 \implies f(x) = 0.$$
Thinking in terms of operations, multiply both sides of the equation by $g(x)$.
You should also consider whether the roots of the numerator are also roots of the denominator.

17. The implication in Pythagoras' theorem goes both ways. So, it can be used to find lengths, or, in this case, to prove that a triangle is right-angled.

18. The definition of a logarithm is as follows: $\log_p q$ translates as "the index you need to raise p by to get q". For example, $\log_2 8 = 3$. So, raise p by $\log_p q$, and what do you get?

19. (a) Use the factor theorem: when $x = p$ is a root, $(x - p)$ must be a factor.
    (b) Substitute the point $(0, 48)$ in.

20. In each case, consider whether there are squares in the formulae, i.e. whether the summary statistic is in the same units as the original data. Three of these are in the original units, one of them isn't.

21. These are a pair of parallel lines, so the points equidistant from them form a third parallel line halfway between the two.

22. Translate the proportionality statements into equations with constants of proportionality a and b. Eliminate p from these to find y in terms of x. Combine the two constants of proportionality into a new constant k. Then substitute in $x = 1$, $y = 5$ to find k.

23. (a) Take out a factor of 10^n, then use the fact that $11 = 1.1 \times 10$.
    (b) Group into 30×10^{2n}. Then transfer a factor of 10 from 30 to 10^{2n}.

24. Rearrange to $y = \frac{1}{x}$ if that form is more familiar. The graph is one of inverse proportion, in both the positive and negative quadrants.

25. Use Newton's Second Law $F = ma$ (NII). Note that the F in $F = ma$ is "resultant force in the direction of the acceleration."

26. The possibility space is a 6×6 grid. Count up the successful outcomes to give a fraction over 36.

27. This is written in complicated language, but is a simple result. Substitute the inputs $(k - x)$ and $(k + x)$ into $f(x) = ax + b$, and simplify. To show that the result is "independent of x" just means showing that it contains no mention of x. The value of x does not affect it.

28. (a) The "boundary equation" has a solution set which forms the boundary of the solution set of the inequality. So, the inequalities $x^2 < 4$ and $x^2 \geq 4$ both have boundary equation $x^2 = 4$.
    (b) Factorise.
    (c) Sketch a positive quadratic, with x intercepts at the roots just found.
    (d) You are looking for the x values at which the y values are greater than zero. In other words, you are looking for locations on the x axis for which the graph is above the x axis. Your answer should be written in set notation.

29. (a) The fraction inside the limit is the gradient of a chord of $y = x^2$. The limit asks "Towards which value does the gradient of the chord tend?"
    (b) The x^2 terms cancel, leaving a common factor of h in the numerator. Since this is non-zero (you haven't taken the limit yet), divide top and bottom by h.
    (c) With no factor of h in the denominator, it is safe to take the limit. Effectively, this means setting h to zero.

30. It's easier to start with the form $4(x^2 + 6x) + 54$. Complete the square on $x^2 + 6x$, then multiply by 4 afterwards.

31. (a) Functions can be many-to-one, and do not have to map to all elements of the codomain.
    (b) Invertible functions must be one-to-one. Ask whether you could determine a unique input in D, given a specific output in C.

32. Since this is a linear inequality, it can be solved algebraically, without any need to sketch graphs. Rearrange to make x the subject, remembering to flip the inequality sign when dividing by -2.

33. "Thrice" is an old word for "three times". Call the first number x, and set up and solve a single equation in x.

34. (a) The bar notation means "evaluated at", so $f(x)\,|_{x=5}$ means plug $x = 5$ into $f(x)$. This is equivalent to $f(5)$, which is more succinct if f is defined. The bar notation is used when there is no f defined, in expressions such as $\sqrt{x^2 - 4}$, $\frac{dy}{dx}$ or xyz.
    (b) State the relevant theorem.

35. Substitute the latter equation into the former. When you rearrange, remember the $\pm$.

36. First, sketch the graph $y = -x^2$, then think about the effect of swapping x and y.

37. (a) Your force diagram should have four forces and an acceleration. Don't include the units.
    (b) Use NII.
    (c) Having found the acceleration, the relevant *suvat* equation is $s = ut + \frac{1}{2}at^2$.

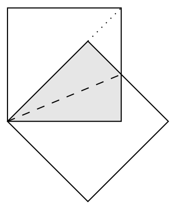
38. (a) There are eight outcomes in the possibility space. List them alphabetically.
    (b) Count the number of successful outcomes in the possibility space. Alternatively, use the binomial distribution.

39. (a) Set $x^2 - x - 6 = 0$ and solve.
    (b) Find the gradient formula $\frac{dy}{dx}$ using
    $$y = x^n \implies \frac{dy}{dx} = nx^{n-1}.$$
    (c) Evaluate $\frac{dy}{dx}$ at the x values in (a).
    (d) Consider the fact that a parabola $y = f(x)$ has $x = k$ as a line of symmetry, for some constant k.

40. Multiply by the denominator $x+a$, gather the x's onto one side, take out a common factor of x, then divide to get x on its own.

41. The boundary equation has been solved correctly. And each individual inequality comparing x to a number is correct. But the combination of the two inequalities isn't.

42. Using the hypotenuse in a $(1, 1, \sqrt{2})$ Pythagorean triangle, show that the side lengths of the kite are 1 and $\sqrt{2}-1$. Then split the kite into two symmetrical right-angled triangles to find its area:



43. Take logarithms of both sides, base 3.

44. Use the facts that
    - the curve is a circle, to find a,
    - the centre is on $y = x$, to find b,
    - the circle goes through the origin, to find c.

45. Square root both sides, remembering a $\pm$. The graph consists of two intersecting straight lines.

46. Δx, or change in x, is another way of expressing the displacement s. This usage, which means change in, is not to be confused with Δ on its own, which signifies the quadratic discriminant $\Delta = b^2 - 4ac$.

47. The roof diagonals have length $\frac{\sqrt{2}}{2}$.

48. You don't need to multiply this out, and shouldn't try! It is already in the form you want. Use the fact that, if two factors multiply to give zero, then one of them must be zero.

49. Consider the *directions* of the forces, relative to each other, which would give the greatest or least possible magnitude of acceleration.

50. Consider one full journey around the perimeter, divided up equally between n vertices.

51. Complete the square once for x and once for y. Gather the constants together, and rearrange to the form $(x-a)^2 + (y-b)^2 = r^2$.

52. To simplify, it's often helpful to write such sums out longhand, e.g. (a) is $\underbrace{1+1+\ldots+1}_{10\text{ times}}$.

53. Having solved to find the roots, sketch the graph of $y = 42x^2 - 71x + 30$ to visualise the problem. The decimal search method relies on being able to discern a sign change, where $42x^2 - 71x + 30$ switches from negative to positive or vice versa.

54. Since $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors, you can use Pythagoras. The velocity triangle is right-angled, with perpendicular lengths 12 and 5.

55. (a) The roots of the boundary equation border the regions which satisfy the inequality.
    (b) This is a negative parabola.
    (c) Consider the x values for which $y = -x^2 - 6x$ is above the x axis.

56. Consider the role of Δ in the quadratic formula.

57. Use $F = ma$, and solve for m algebraically.

58. Multiply out before differentiating. The general result is that, if $\text{g}(x) = kx^n$, then $\text{g}'(x) = nkn^{n-1}$.

59. When working with small numbers of items, there's no harm in listing explicitly until you have internalised the relevant combinatorics formulae.

60. There are 2π radians in a circle, or, equivalently, π radians in a semicircle or along a straight line.

61. Draw a sketch with A and B at arbitrary locations. (This is often helpful in vector problems.)
    (a) Construct the journey from A to B as: back from A to O, and then forwards from O to B.
    (b) The position vector the midpoint of A and B is the average of the position vectors of A and B.

62. Use the factor theorem: if $x = \alpha$ is a root of a polynomial, then $(x - \alpha)$ is a factor.

63. The relevant concept is independence: does the fact that my birthday falls on a Tuesday this year affect the probability that it does next year too?

64. Factorise as $(x^2 + \ldots)(x^2 - \ldots) = 0$.

65. Sketch a triangle ABC, and drop a perpendicular from the point B to the side AC. Find the height in terms of length a and angle C.

66. (a) The notation gf means "apply f then g".

(b) f^{-1} is the inverse of f, mapping back from output to input.
(c) f^2 is the application of f twice in succession, feeding the output back in as a new input.

67. Use the fact that $\log_x y$ stands for "the power you need to raise x by to get y".

68. Put the two fractions of the RHS over a common denominator. Then look for a common factor on the top and bottom.

69. Rewrite as $\pm(2x-1)=9$.

70. Vertical asymptotes are lines of the form $x=k$, at which the value of a function or expression tends to either plus or minus infinity. Such a vertical asymptote appears where there is division by zero in the algebra.

71. Draw a clear diagram and use the cosine rule.

72. This is an immediate result: there's no need to use any calculus. You could replace $y=x^2-x$ with any other polynomial curve $y=\mathrm{f}(x)$ and the answer would be the same.

73. Work out the number of turns n involved in the shortest possible route. To take the shortest route, the correct direction must be chosen at each of these, meaning that the probability is $\frac{1}{2}^n$.

74. (a) Consider whether $y=7$ is a minimum or a maximum.
(b) To find the minimum point (the vertex), set $\mathrm{f}'(x)=0$ or complete the square.
(c) Set up the equation $\mathrm{f}\left(-\frac{2}{a}\right)=0$ and solve.

75. Sketch the lines, and consider the fact that a derivative with respect to x is a rate of change *as x changes*. In only one of these equations is x allowed to change.

76. Use the factor theorem, and consider the three equations it generates one by one.

77. The notation $\Big[\mathrm{F}(x)\Big]_a^b$ means $\mathrm{F}(b)-\mathrm{F}(a)$.

78. Remember that the mean is a measure of central tendency, while the variance is a squared measure of spread.

79. A counterexample is an example that disproves a statement. You don't have to be picky: any one is as good as any other. So, in each case, find any element of the former set that is not an element of the latter.

80. Multiply top and bottom by the conjugate of the denominator, i.e. $\sqrt{7}+2$.

81. Use Pythagoras (forwards implication) to find the side lengths of the triangle. Then use Pythagoras (backwards implication) to show that $\triangle AXC$ is right-angled.

82. Look for a counterexample: a rational number with a non-terminating decimal expansion.

83. Substitute into the LHS, and use a Pythagorean identity.

84. Count successful and total outcomes, and use the rule, for equally likely outcomes: $p=\frac{\text{successful}}{\text{total}}$.

85. Consider the average rate of change of y with respect to x. This means $\frac{\Delta y}{\Delta x}$, which would be constant in a linear relationship.

86. Use the factor theorem. Then show that one factor produces no roots, and the other produces one.

87. Find the times taken t_A and t_B, in seconds, for each to run the race. The answer is t_B-t_A.

88. Remember that $x-p\equiv-(p-x)$.

89. Use Pythagoras to calculate the squared distances from each to $(1,0)$.

90. Remember the definition of a logarithm: $\log_a b$ is "what you have to raise a by to get b".

91. $\frac{dy}{dx}$ is the gradient. You don't need calculus.

92. (a) Factorise or use the formula.
(b) Sketch a positive parabola with single roots as calculated in part (a).
(c) The solution set lies between the roots.

93. There are 2π radians in 360°.

94. Draw in the radii from the centre of the semicircle to the points of intersection.

95. The vector $\mathbf{r}$ stands for a generic position vector, so the equation, written more fully, is

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-3\\1\end{pmatrix}+\begin{pmatrix}2\\5\end{pmatrix}t.$$

You can read the gradient straight from the second term; technically, you'll be using the formula

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

Then use $(-3,1)$ in $y-y_0=m(x-x_0)$.

96. No calculation is needed. Join the midpoints of the edges with lines parallel to the sides of the square. Then count triangles.

97. Calculate $\sum x$ (the sum of all the x values) for each set, by rearranging the formula for the mean. Then add the two values of $\sum x$ and divide by $n_1 + n_2$ to find the combined mean.

98. Dissect and rearrange a general trapezium to make a rectangle.

99. (a) Consider negative integers.
    (b) Consider $0 < a < b < 1$.

100. In (a) and (c), the answer comes immediately from $F = ma$. Only (b) requires a bit of calculation. Since the forces are perpendicular, the resultant is the Pythagorean sum of the magnitudes.

——— End of 1st Hundred ———